

# A reliable Evaluation Method for 2D Wireless Sensor Network Deployment

Athanasios Iliodromitis, George Pantazis, Vasileios Vescoukis

**Abstract**—In recent years, Wireless Sensor Networks (WSNs) have rapidly evolved and now comprise a powerful tool in monitoring and observation of the natural environment, among other fields. The use of WSNs is critical in early warning systems, which are of high importance today. In fact, WSNs are adopted more and more in various applications, e.g. for fire or deformation detection. The optimum deployment of sensors is a multi-dimensional problem, and has two main components; network and positioning approach. Lots of work has dealt with the issue, and one can find several solutions for sensor deployment either achieving ideal network topology or achieving fine geometry. In most cases, it is hard or even impossible to achieve perfect geometry in nodes' deployment. The desirable scenario of nodes arranged in square or triangular grid would raise extremely the cost of the network, especially in unfriendly or hostile environments. However the user wants to know how much does the deployment plan, approximates the ideal geometry. A geometry as near as possible to the ideal one, minimizes the numbers of sensors needed, which subsequently means less costs for the entire network. This paper suggests an evaluation method for 2D Wireless Sensor Networks, concerning the geometry of the distributed sensors, in large scale WSNs. It approximates the solution comparing the random TIN that the nodes of the network form, with the ideal geometry, i.e. the regular equilateral triangle grid. The user knows a priori, how much does the proposed deployment plan, achieves a deployment as near as possible to the ideal one, for the given characteristics of the study area.

**Index Terms**—Centroidal Voronoi Tessellation, CVT, Delaunay Triangulation, sensors, sensor deployment, spatial coverage, Wireless Sensor Network, WSN.

## 1 INTRODUCTION

RECENTLY, Wireless Sensor Networks (WSNs) have rapidly evolved and now are a powerful tool for monitoring and observation of the natural environment, among other fields. The use of WSNs is critical in early warning systems, which are of high importance today. In fact, WSNs are adopted more and more in various applications, e.g. for fire or deformation detection.

Lots of work has been done for the optimization of such networks and concern both geographical and network coverage. Full geographical coverage of an area is meaningless, if communication between the sensors is weak or the energy consumption is extremely high. On the other hand, even if the connection problems are solved, the network fails its mission if geographical coverage is not achieved.

Algorithms for geographical coverage can be found, but in most cases it is hard or even impossible to achieve perfect geometry in nodes' deployment. The desirable scenario of nodes arranged in square or triangular grid would raise extremely the cost of the network, especially in unfriendly or hostile environments. However the user wants to know a priori how well does the deployment plan approximates the ideal geometry. A geometry as near as possible to the ideal one, minimizes the numbers of sensors needed, which leads to less costs for the entire network.

In this paper an evaluation method for 2D Wireless Sensor Networks, concerning the geometry of the distributed sensors, in large scale WSNs is suggested. It approximates the solution comparing the random triangulation irregular network (TIN) that the nodes of the network form, with the regular equilateral triangle grid.

The proposed method is based on the Delaunay Triangulation and the properties of the triangle geometry. An evaluation index is proposed. The index takes into consideration the sensing range of the sensor, in order to provide the user with information and statistics for the achieved geometry, so that he can feedback the system and create scenarios as near as possible to the ideal geometry [1], [2].

The rest of this paper is organized as follows: In Section 2, previous works related to the evaluation of area coverage methods are described. In Section 3, the importance of the evaluation of a deployment plan is emphasized. In section 4, possible indexes for the evaluation of a 2D WSN, concerning the coverage are given and the suggested methodology is thoroughly explained. In section 5 explains briefly the triangulation theory that is related to the index proposed in section 4. In section 6 examples based on the proposed methodology are given. Finally, Section 6 concludes the paper.

## 2 RELATED WORK

The scientific community has been occupied to a significant extent with finding the optimum solution for deployment of WSN nodes. Algorithms for optimizing both network communication and geographical coverage can be found in many papers and variations. Apart from the optimization part, a comprehensive solution must include an evaluation index to

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verify how good or how efficient the deployment is. One can find many proposals for evaluation indexes related to optimization of network communication between sensors or minimization of energy consumption [3], [4], [5].

However most of the indexes related to the geographical coverage compare the proposed deployment method with other deployment methods, without providing a universal index for the spatial distribution of the sensors of a wireless sensor network [6].

A typical index is proposed by Chizari, Hosseini, & Poston, in their work [7]. They determine the percentage of the area covered by at least one sensor in relation to the whole area and the distances between the sensors. Furthermore, the sensors are separated in those that have large, adequate or small number of other sensors near them. The percentage of the supervised area is also used as an index by Vieira, Vieira, et al. [8]. In both cases no information is given for the sub-areas that are not covered by any sensor or how well the deployment of the sensors approximates the ideal triangular grid, which results in the optimum area coverage.

A different solution is given by Kolega [9], [10], [11]. There a deployment method using a clustering technique is proposed. A regular triangular grid is overlapped to the final deployment positions and if the distance between the theoretical and the actual position is less than a threshold, the sensor is considered to be unmoved. The index is the number of the sensors that coincide the theoretical positions. In other words, the less sensors are kept unmoved, the more successful is the deployment plan.

### 3 THE IMPORTANCE OF THE EVALUATION OF A DEPLOYMENT PLAN

As mentioned previously, one can find plenty of algorithms for sensor deployment in a WSN. As far as the geographical coverage, the final solution of the problem depends on a number of parameters and constraints, such as the number of points to be observed, their position, the sensing range of the sensors, the number of them (if this is limited) etc.

Each solution apart from the final deployment positions should also provide statistical indexes, indicating the need for a different scenario, e.g. different number of sensors or another type of sensor (which implies a different sensing range).

More specifically, the percentage of the points that are not observed by any sensor, is a key factor for adding more sensors, depending on the needs of the application and the desired coverage. The percentage of those being supervised by two or more sensors is related to the network reliability. In many applications it is critical that each point (or a percentage of them) to be in the sensing range of more than one sensors.

The ideal node geometry for a WNS for large-scale applications is when all the sensors are arranged in a grid of equilateral triangles. The greatest advantage of the grid of equilateral triangles is that each point is equidistant from all adjacent. Thus, the maximum coverage is achieved, using the minimum number of sensors [1], [2], [9].

In the special occasion where the points to be observed are dense and evenly distributed, the final deployment solution

would, indeed, consist of nodes deployed on the vertexes of equilateral triangles. In any other occasion the points will form a triangulation irregular network (TIN) (fig. 1).

As demonstrated in this document, the numbering for sections upper case Arabic numerals, then upper case Arabic numerals, separated by periods. Initial paragraphs after the section title are not indented. Only the initial, introductory paragraph has a drop cap.

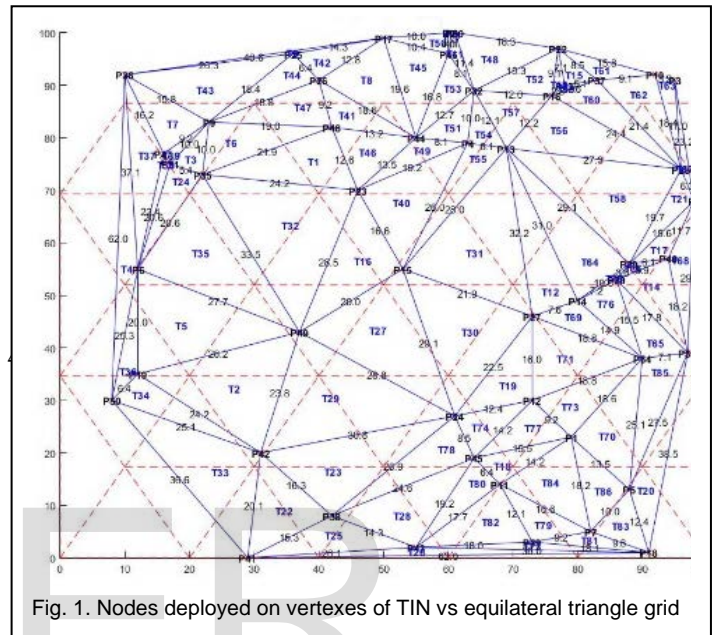


Fig. 1. Nodes deployed on vertexes of TIN vs equilateral triangle grid

The question arising, is whether the deployment solution resulting from the used methodology (whichever this is), how close is to the ideal solution. Moreover, every solution should be followed by its evaluation. This way the user can understand how efficient the different scenarios are, and if necessary create new scenarios for improved geometry.

In next section, possible indexes for the evaluation of a 2D WSN. The properties of each are analyzed and the advantages and disadvantages of each are written down. Among them, the one that describes better the relationship between the theoretical and the proposed final solution is selected.

### 4 POSSIBLE INDEXES FOR WSN DEPLOYMENT EVALUATION

In limited field applications where the sensors to be deployed are few, one can choose the positions so that the sensors form the desired equilateral grid. But there are cases, especially in large scale applications, where hundreds or thousands of sensors have to be deployed, that the positions must belong to the original set. Therefore result additional constraints [1]. For example in a case of an application for fire detection in a forest, where the trees are in random positions, sensors can only be placed in a sub-set of the dataset consisting of all the tree positions.

#### 4.1 The Clark – Evans Index

The science of spatial analysis and the geographic information

systems provide indexes for evaluating a spatial distribution, such as the deployment in a wireless sensor network.

The spatial dispersion of point distributions can be determined by the index D [12]. Using this index, there is a scale created, starting from the aggregate pattern, continues to random pattern and ends up to the uniform spatial pattern.

Practically, the existing spatial distribution is compared to the theoretical distribution. Let a surface with area A, containing N points. Assuming a random spatial distribution, it has been proven that the probability for a point to exist at a distance d, follows the normal distribution with mean  $d_a$  and is given by (1):

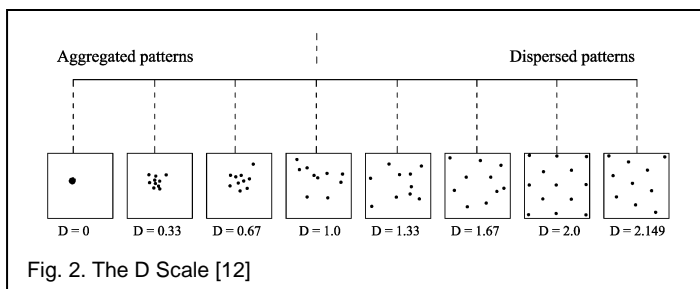
$$d_a = \frac{1}{2} \cdot \sqrt{N/A} \tag{1}$$

Since the mean distance of each point from its nearest points, is given by (2):

$$d_\pi = \frac{1}{N} \cdot \sum_{i=1}^N d_i \tag{2}$$

This value is a sample value of the theoretical distribution. The index  $D = d_\pi/d_a$ , is defined. The values of the D scale, as it are known, ranging from 0 to 2.149 [13]. When  $D = 0$ , the points are all in the same position (aggregated pattern). When  $D = 1$  points are randomly distributed. Finally, high values of D represent varying types of dispersion. The maximum value  $D = 2.149$  refers to the hexagonal uniform spatial pattern. (fig. 2)

This method is very good to demonstrate how well the distribution of the sensors approaches the ideal deployment, i.e. regular hexagons (or respectively equilateral triangles), but has a major drawback. It is an index concerning the whole area and therefore fails to show in which part of the area there are weaknesses and deficiencies, so that this specific sub-area be strengthened with additional sensors. Consequently, the appropriate index should be based on geometric features of the existing deployment.



### 4.2 The area and perimeter of the triangle

Using the area of a triangle as an evaluation index, the area of an equilateral triangle, with edge length R (where R is the range of the sensor, in the specific application), is compared to the area of a random triangle results from the coordinates (x, y) of its vertices (Gauss's formula):

$$E_{\text{equilat}} = \frac{\sqrt{3}}{4} \cdot R^2 \tag{3}$$

$$E_{\text{random}} = \frac{1}{N} \cdot \sum_{i=1}^3 (x_{i+1} - x_{i-1}) \cdot y_i \tag{4}$$

This index is inappropriate for the evaluation of the desired solution. A random triangle may have the same area as an equilateral triangle with edge length R, but its vertices can be in such positions that be too far from the ideal geometry. [1]

Using the perimeter as an index, it is checked whether the pe-

rimeter of a random triangle with edge lengths a, b, and c approximates the sum of the edges of an equilateral triangle with edge length R, (i.e. 3·R).

$$P_{\text{equilat}} = 3 \cdot R \tag{5}$$

$$P_{\text{random}} = a + b + c \tag{6}$$

Respectively to previous case, the perimeter of the triangles is unsuitable as an index, as a random triangle may have the same perimeter with an equilateral triangle with edge length R, but the distance of its vertices be prohibitive for the area coverage between them. [1]

### 4.3 Standar deviation of the mean of the triangle edges

The fact that the equilateral triangle consists of three edges equal, leads to a specific property, using measures of dispersion from the science of statistics [15]. The standard deviation of the mean of its edges, is equal to zero. For an equilateral triangle with edge length R (where R, the sensing range of the sensor), the mean length of its edges is:

$$\hat{x} = \frac{R+R+R}{3} = R \tag{7}$$

Respectively, the standard deviation is:

$$\sigma_0 = \frac{\sqrt{[uu]}}{n-1} = \frac{0}{2} = 0 \tag{8}$$

For any random triangle mean and standard deviation are:

$$\hat{x} = \frac{a+b+c}{3} \tag{9}$$

$$\sigma_0 = \frac{\sqrt{[uu]}}{n-1} \tag{10}$$

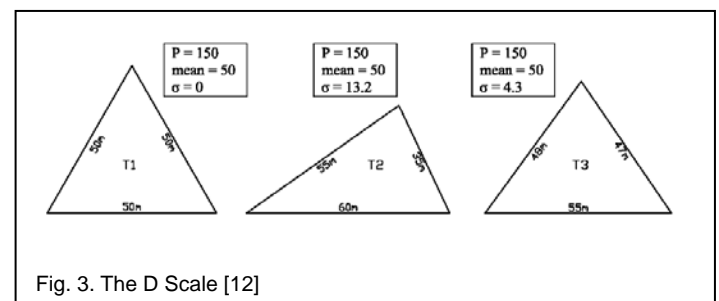
Where:

a, b, c: the length of the three edges.

u = the difference of each edge from the triangle mean,

n = 3, the number of triangle edges

Using the standard deviation of the mean of the triangle edges as index for any random triangle, it applies that the smaller the standard deviation is, the closer the random triangle is to the equilateral one. As shown in figure 3, two random triangles can have the same perimeter P, and/or the same mean length edge. Even so, the one that best approximates the equilateral triangle is the one with the smaller standard deviation (fig. 3).



#### 4.3.1 Derived statistical indexes

A regular triangular grid, consists of equilateral triangles, equal to each other (figure 1). Based on the property that was analyzed previously, the mean of the standard deviations of the means of triangle edges will be equal to zero (as all individual standard deviations are equal to zero).

Thus, a triangulation which has resulted from a random set of points (TIN) can be compared with the regular triangular grid,



using as an index the mean of the standard deviations of each individual triangle. The smaller this value is, the more the TIN will approximate the regular grid.

Another advantage of the above is that the index is not affected by the displacement or the rotation of the TIN irregular grid of triangles with respect to the regular grid, and only depends on the size of the side of each triangle. The geometry that is formed by the irregular network is compared directly to the geometry of the regular triangle grid.

Simultaneously with the calculation of the mean of the standard deviation of the grid, three other statistical indexes should be examined: The minimum value of the standard deviations should be close to zero. On the other hand, a high maximum value, shows that there are triangles in the mesh that create bad geometry. Useful conclusions can result from the median of the standard deviations.

Finally, the frequency histogram of the standard deviations should be taken into consideration. The more standard deviations are gathered at low prices, the more the TIN approaches the regular one.

#### 4.3.2 The $\sigma_0/R$ index

The use of the mean value of the standard deviations of the edges of each triangle of the TIN as an index, provides useful conclusions about how well the triangles that are formed adapt the regular triangular grid. So when the number of the triangles is increased or decreased (e.g. when the number of sensors changes, respectively), the index changes and the results are directly comparable.

The problem arises when comparing scenarios involving sensors with a different radius range. It is logical that the greater the range of the sensor is, the higher prices the standard deviations of the edges of the triangles will get.

On the other hand, in two scenarios where  $R_{s1} = 50m$  and  $R_{s2} = 20m$  where  $\sigma_{1,average} = \sigma_{2,average} = 2m$ , the standard deviation indicates better adjustment in the first case than in the second. Thus, when it comes to comparing scenarios with sensors of different sensing range the index to be used will be:

$$g = \bar{\sigma}_0/R \quad (11)$$

The smaller prices the index gets, the better adjustment is achieved.

## 5 EVALUATION ALGORITHM BASED ON DELAUNAY TRIANGULATION

After choosing the appropriate index, it is obvious that the evaluation algorithm is based on triangulation of the point dataset, which correspond to the final sensor deployment positions. The Delaunay triangulation is a triangulation of a point dataset, which meets specific properties. To understand these properties, some elements from the triangulation theory are given.

Let  $P:=\{p_1, p_2, \dots, p_n\}$  be a set of points in the plane and a maximal planar subdivision  $S$ , such that no edge is connecting two vertices without destroying its planarity, meaning that any edge that is not in  $S$  intersects one of the existing edges. A triangulation of  $P$  is defined as a maximal planar subdivision whose vertex set is  $P$  [16]. The same definition differently

worded is given in [17]: "A triangulation of a planar point set  $P$  is a subdivision of the plane determined by a maximal set of noncrossing edges whose vertex set is  $P$ ".

The Delaunay triangulation is a special case of triangulation and is the dual problem of Voronoi polygons. It owes its name to the Russian mathematician Boris Nikolaevich Delaunay, after researching the subject in 1934 [16]. For a point set  $P$ , a Delaunay triangulation of  $P$ , is a triangulation that only has legal edges. (An edge is illegal if locally the smallest angle can be increased by flipping that edge).

There are a number of algorithms for creating Delaunay triangulation. Just to name a few: flip algorithms, incremental algorithms, "divide and conquer" algorithms and the sweepshull algorithm [18], [19].

The Delaunay triangulation due to its properties has been used in various deployment methods in WSNs [5], [20], [21]. In this paper its properties are used for the evaluation of any method used for deployment.

For the proposed methodology, the input parameters are the coordinates of the deployment positions and the sensing range. For these points, the corresponding Delaunay triangulation is generated. To each triangle created, the three points that compose it and the corresponding edge lengths are assigned. Additionally, the standard deviation of the mean of the edges for each triangle is calculated.

The user can check if there are undesirable length values (extremely high values) or triangles with high standard deviations values (usually triangles generated in the outer boundary of the area). High standard deviation of a triangle means that its edges have completely different lengths. Thus, it is possible to interfere with non-automated process and deploy sensors, to improve the geometry (or at least the coverage) locally.

The mean of all of the standard deviations of the triangles and the index  $\sigma_0/R$  index are calculated. This value is the index of the problem and is used to compare the different scenarios created or the different methodologies used for the deployment. If it is not satisfactory, the parameters of the problem (number of sensors, sensing range) are redefined and the deployment process repeated.

Finally, the other statistical indexes that calculated (maximum and minimum standard deviation and the median) can be used together for the evaluation of the deployment plan.

## 6 CASE STUDY

There were scenarios with different number of points, areas of different dimensions and various sensing range  $R_s$  in order to test the proposed methodology. For the choice of the deployment positions, the algorithm described in [1], [2] was used, making used of the cendroidalvoronoi tessellation (CVT) [22], [23].

In the images that follow an area with dimensions 500mx500m is presented and two basic scenarios were created.

For the first basic scenario with 2000 points for observation and sensing range  $R_s=40m$  (fig.4) are presented the deployment solution, the TIN created and the frequency diagram for  $N=50$  sensors (fig. 4a, 4b, 4c),  $N=60$  sensors (fig. 4d, 4e, 4f) &

N=70 sensors (fig. 4g, 4h, 4i).

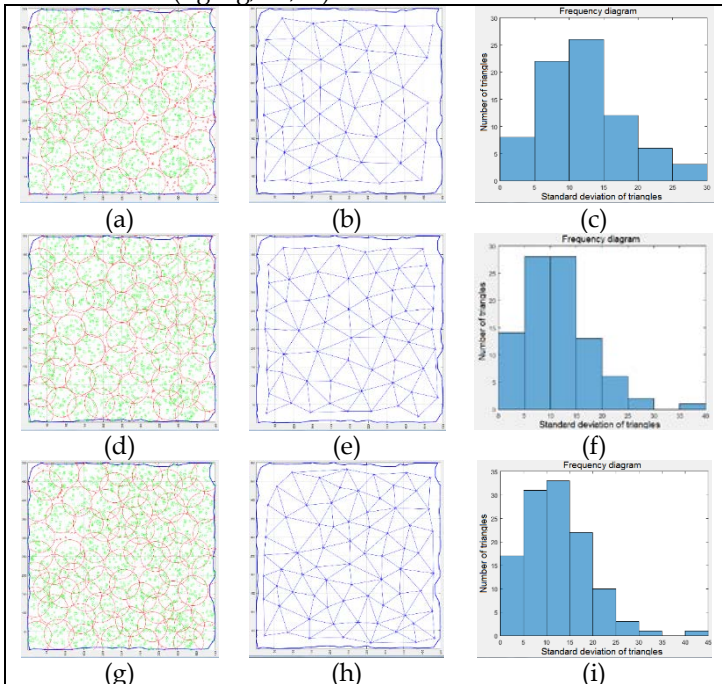


Fig. 4. The different scenarios that were created for Rs=40m

Respectively, for the second basic scenario with 2000 points and sensing range Rs=20m (fig. 5), are presented the deployment solution, the TIN created and the frequency for N=200 sensors (fig. 5a, 5b, 5c), N=240 sensors (fig. 5d, 5e, 5f) & N=280 sensors (fig. 5g, 5h, 5i). Table 1 summarizes the results.

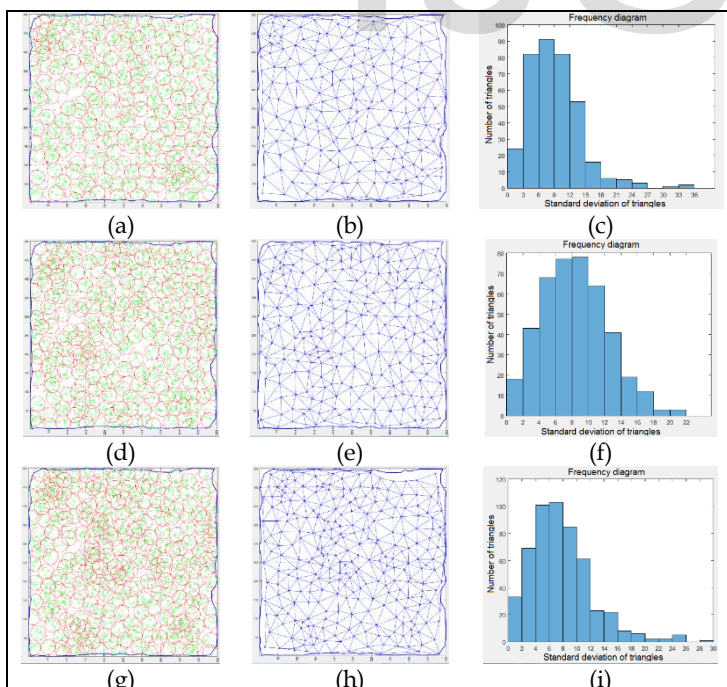


Fig. 5. The different scenarios that were created for Rs=20m

TABLE 1

THE RESULTS FOR THE CREATED SCENARIOS

	Number of sensors	$\sigma_0$ (m)	$\sigma_0/R$	mean	median	Max
Rs=40m	50	12.46	0.311	14.21	14.25	34.48
	60	11.48	0.287	11.65	11.78	25.11
	70	11.41	0.284	10.29	10.84	21.13
Rs=20m	200	8.38	0.464	8.47	8.09	26.11
	240	8.13	0.420	8.09	7.74	23.65
	280	7.72	0.394	7.99	7.20	22.18

## 7 CONCLUSIONS

Wireless Sensor Networks (WSNs) are increasingly used the last few years to support a wide variety of applications, such as environmental monitoring, structural monitoring, security detection, just to name a few. In all cases apart from the need for network operation, full geographical coverage is required.

One important factor is to deploy the sensors in a manner that will achieve full coverage with the minimum number of nodes. Any method used for the deployment, it should be followed by an evaluation index. Then the different scenarios (or different methods) are comparable and money can be saved by using less sensors, placed at more efficient positions.

This paper proposes an algorithm for evaluating the geometry achieved in a wireless sensor network based on Delaunay triangulation and on triangle geometry. The deployment positions are modeled as a Delaunay triangulation and all scenarios (different number of sensors or different sensing range) are compared to the equilateral triangle grid.

The proposed methodology is easy to be programmed as it is based on tools and methods of computational geometry.

As an index, the ratio of the mean of the standard deviations of the triangle edges to the sensing range ( $g$ ) is used. The proposed index takes into account the geometry of triangles that are formed from the deployment positions. It is independent of the position or the orientation of the triangles.

Furthermore, it allows to compare the triangles meshes created in scenarios with different sensing range. The metric is unique for each scenario, so the different scenarios are directly comparable.

The other statistical indexes provided along, (minimum and maximum value, median) can help the user in the decision of changing the parameters of the scenario or focus in specific sub-areas with bad geometry and deal with them.

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